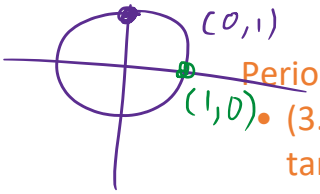
Warm-up!

$\tan\theta = \frac{\sin\theta}{\cos\theta}$



Periodic Graphs Day 6:

- (3.8+3.11) Graphs of tangent and cotangent
- (3.11) Graphs of secant and cosecant

1. List 4 values of θ for which $\tan(\theta)$ is undefined.

$\theta = \frac{\pi}{2} + \pi k$ $\quad \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$

2. List 4 values of θ for which $\tan(\theta) = 0$.

$\theta = \pi k$ $\quad 0, \pi, 2\pi, 3\pi, \ldots$

3. List 4 values of θ for which $\csc(\theta)$ is undefined.

$\theta = \pi k$ $\quad 0, \pi, 2\pi, 3\pi, \ldots$

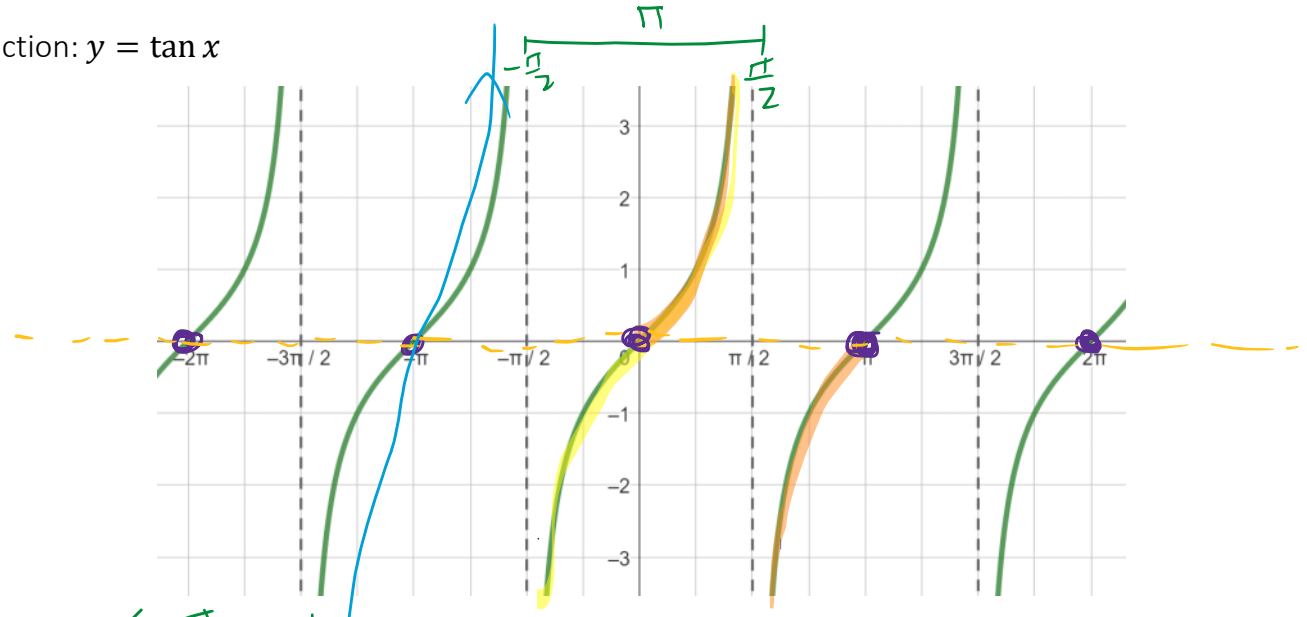
HW: The Other Four Trig Graphs

4. List 4 values of θ for which $\csc(\theta) = 0$.

No such θ.

$\csc\theta = \frac{1}{\sin\theta}$

Parent Function: $y = \tan x$



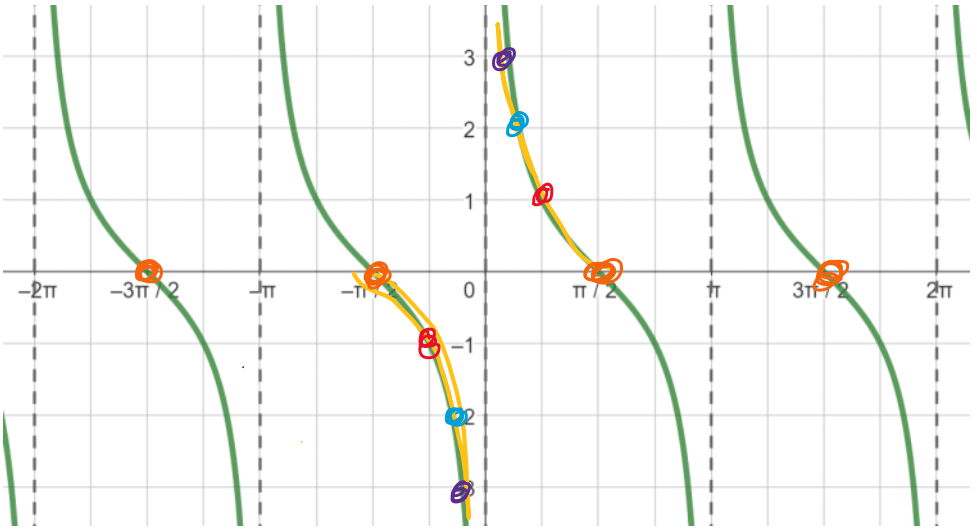
Domain: $x \neq \frac{\pi}{2} + \pi k, k \in \mathbb{Z}$ Range: $(-\infty, \infty)$ Period: π

Between consecutive asymptotes, this function is: increasing

Points of Inflection: $(\pi k, 0)$ Even or Odd:

Mental check we can use to make sure we got the zeros and asymptotes correct:

Parent Function: $y = \cot x$



Domain: $x \neq k\pi$ Range: $(-\infty, \infty)$ Period: π

Between consecutive asymptotes, this function is: decreasing

Points of Inflection: $\left(\frac{\pi}{2} + \pi k, 0\right)$ Even or Odd:

Mental check we can use to make sure we got the zeros and asymptotes correct:

Transforming the Functions

$y = a\tan\big(b(x+c)\big) + d$

a

b Period = $\frac{\pi}{b}$

c

d ← All of the POI are on the line $y = d$

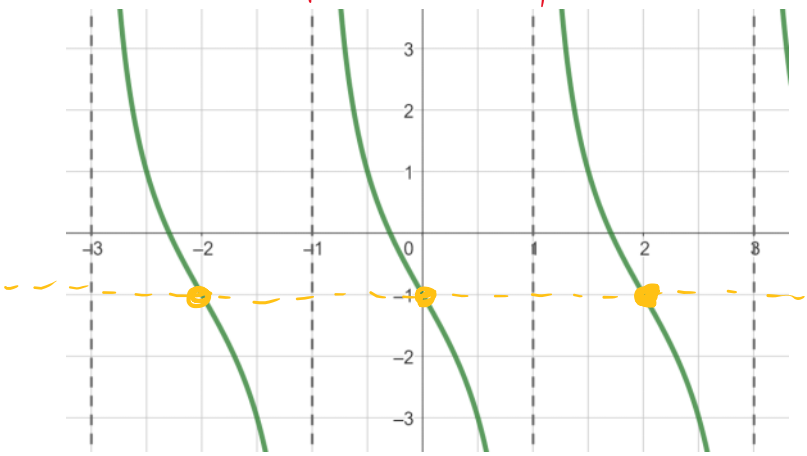
For tangent, the phase shift helps you find a point of inflection.
For cotangent, the phase shift helps you find a vertical asymptote.

Roughly sketch $y = \tan\left(\frac{1}{8}(x + \pi)\right) - 3$

Roughly sketch $y = 4\cot\big(\pi(x - 3)\big) + 2$

You try: Roughly Sketch $y = \tan\left(\frac{\pi}{4}(x+1)\right) + 5$

To the right is a graph of $y = f(x)$. Which is a correct equation for this function?



A) $f(x) = 2\cot\left(\frac{\pi}{2}(x-1)\right) + 1$

B) $f(x) = 2\cot(\pi(x-1)) - 1$

C) $f(x) = 2\cot\left(\frac{\pi}{2}(x+1)\right) - 1$

D) $f(x) = 2\cot(\pi(x+1)) + 1$

$b = \frac{\pi}{\text{Period}}$ $\quad$ $\text{Period} = \frac{\pi}{b}$

$b = \frac{\pi}{2}$

At what x-values does this function have vertical asymptotes?

$x = \ldots, -3, -1, 1, 3, \ldots$

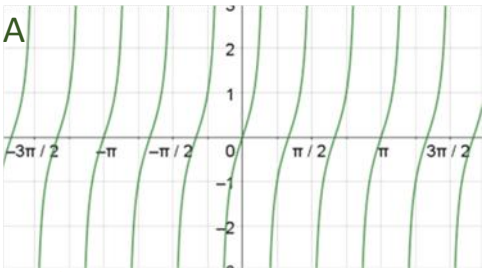
$x = 1 + 2k$

What is the domain of this function?

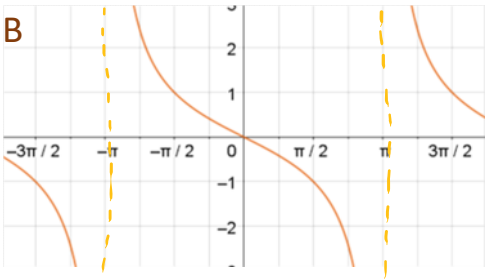
$x \neq 1 + 2k$

Match each of the following functions with the letter corresponding to the correct graph

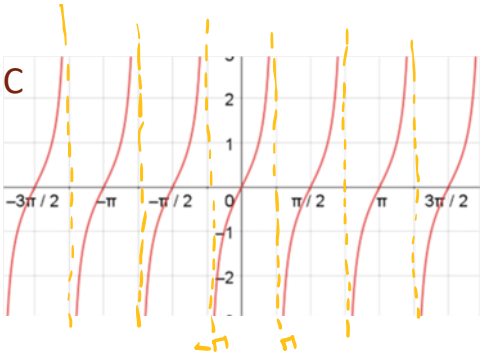
A



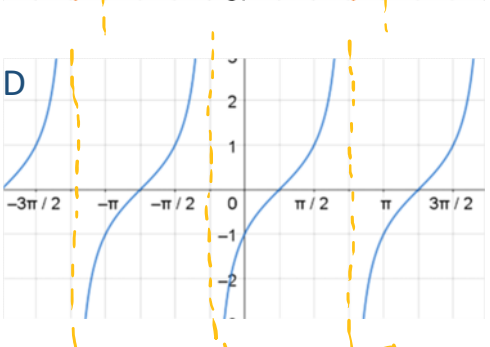
B



C



D



$\frac{\pi}{\pi/2} = 2$ Period $= \frac{\pi}{2}$ $-\frac{\pi}{4}$ $\frac{\pi}{4}$ $-\frac{\pi}{4}$ $\frac{3\pi}{4}$

$f(x) = \tan(3x)$ A

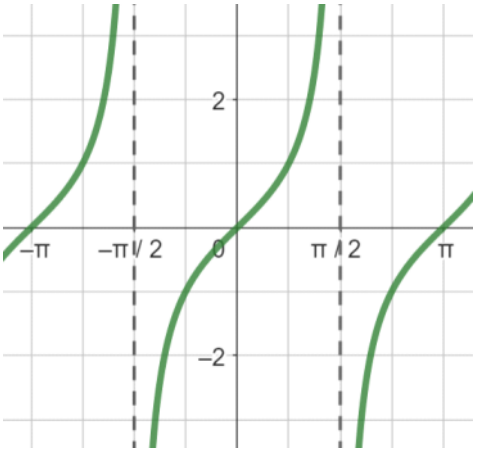
$g(x) = \tan\left(x - \frac{\pi}{4}\right)$ D

$h(x) = \tan(2x + 2\pi)$ C

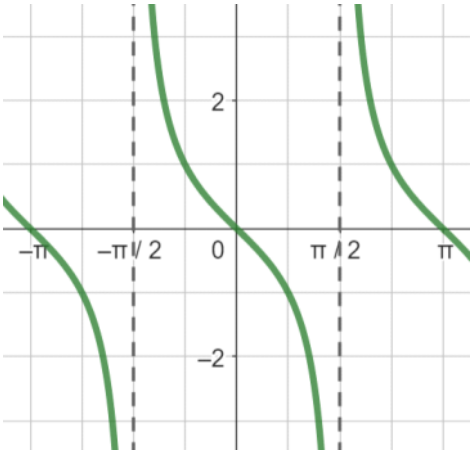
$j(x) = \tan\left(-\frac{x}{2}\right)$ B

Reflecting Tangent

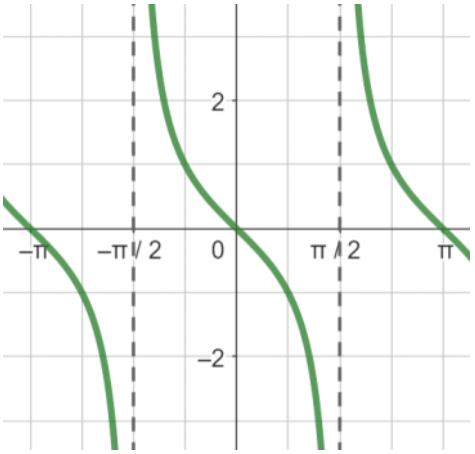
$y = \tan(x)$



$y = -\tan(x)$



$y = \tan(-x)$



$\tan(-x) = -\tan(x)$

Let's make sure we can find asymptotes

Find the vertical asymptotes for $f(x) = 3\cot\left(\frac{1}{4}(x-\pi)\right)+1$

$\pi k = \frac{1}{4}(x-\pi)$

$4\pi k = x-\pi$

$\pi + 4\pi k = x$

$x = \pi, 5\pi, 9\pi, 13\pi, \ldots$

$\frac{1}{4}(x-\pi) = \pi$

$x-\pi = 4\pi$

$x = 5\pi$

Period $= \frac{\pi}{1/4} = 4\pi$

$x = 5\pi + 4\pi k$

You try finding asymptotes

Find the vertical asymptotes for $f(x) = -2\tan\left(2\pi\left(x+\frac{1}{4}\right)\right) - 3$

$$\frac{\frac{\pi}{2} + \pi k}{2\pi} = \frac{2\pi\left(x + \frac{1}{4}\right)}{2\pi}$$

$$\frac{1}{4} + \frac{k}{2} = x + \frac{1}{4}$$

$$\frac{k}{2} = x$$

Which of the following are vertical asymptotes of the given function?

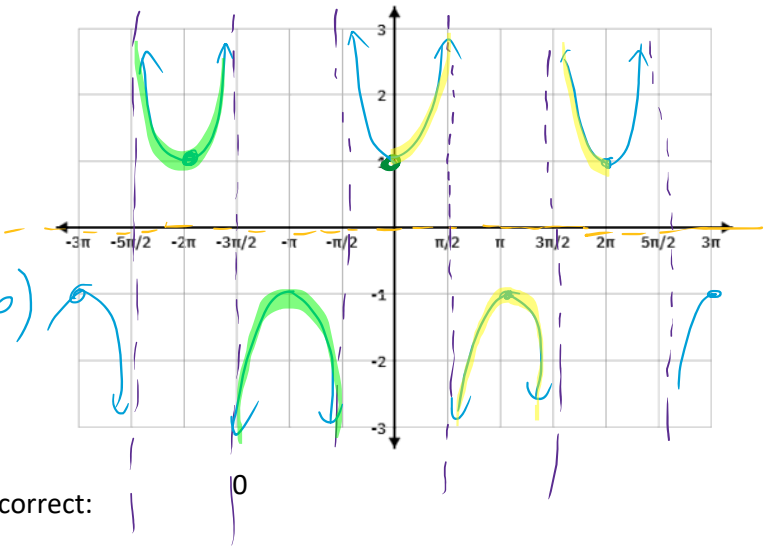
Let $f(x) = 3\tan\left(\frac{1}{4}(x-\pi)\right) + 1$

$x = \pi$ $x = -\frac{\pi}{2}$ $x = 4\pi$ $x = -\pi$

$x = \frac{3\pi}{2}$ $x = 3\pi$ $x = 6\pi$ $x = -5\pi$

Graphing Secant $\sec(x) = \frac{1}{\cos x}$

1) Pretend it is the reciprocal function
2) Put vertical asymptotes at the midline points
3) Put U shapes at the relative maxima and relative minima



Domain: $x \neq \frac{\pi}{2} + \pi k$

Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

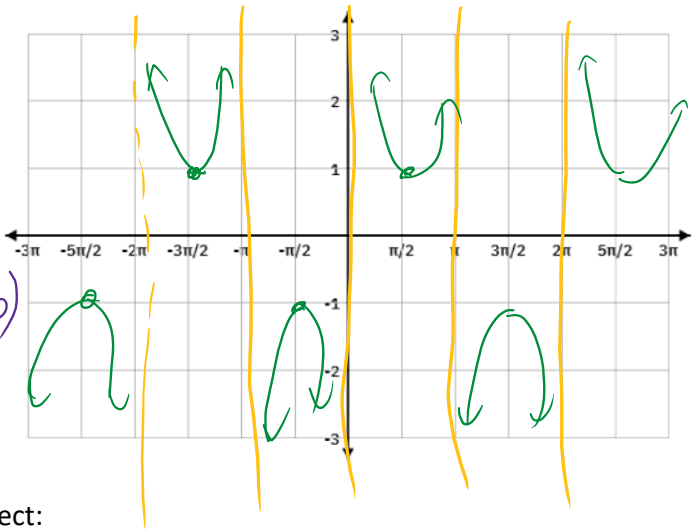
Even or Odd: even

Mental check we can use to make sure we got the asymptotes correct:

0

Graphing Cosecant

1) Pretend it is the reciprocal function
2) Put vertical asymptotes at the midline points
3) Put U shapes at the relative maxima and relative minima



Domain: $x \neq k\pi$

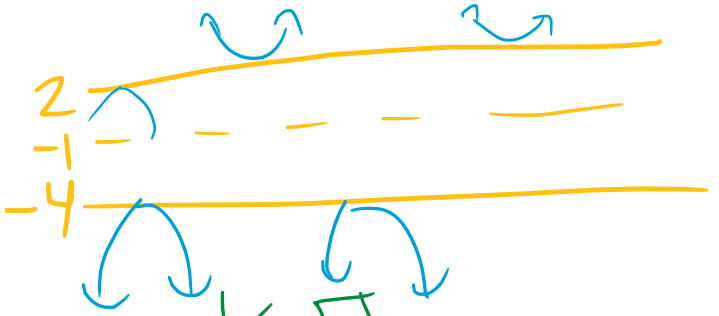
Range: $(-\infty, -1] \cup [1, \infty)$

Period: 2π

Even or Odd:

Mental check we can use to make sure we got the asymptotes correct:

Roughly sketch $y = 3\csc\left(2\left(x + \frac{\pi}{4}\right)\right) - 1$



Range: $(-\infty, -4] \cup [2, \infty)$

$2\left(x + \frac{\pi}{4}\right) = k\pi$

$x + \frac{\pi}{4} = \frac{k}{2}\pi$

Asymptotes: $x = -\frac{\pi}{4} + \frac{k}{2}\pi$

Domain: $x \neq -\frac{\pi}{4} + \frac{k}{2}\pi$

The only things I will ask you about with secant and cosecant graphs

1) Domain/Asymptotes

2) Range

$$f(x) = 2\sec\left(\frac{\pi}{4}(x-1)\right) - 5$$

Find the range

Find the equations of four vertical asymptotes

$g(x) = -3\csc(x + \pi) + 6$

Find the range

Find the equations of four vertical asymptotes